Issue 4.6

Nosokinetics News

December 2007

Bi-monthly Newsletter of the UK Nosokinetics Group.
On line full text and archive at http://www.iol.ie/~rjtechne/millard/index
Previous issues in pdf at www.nosokinetics.org

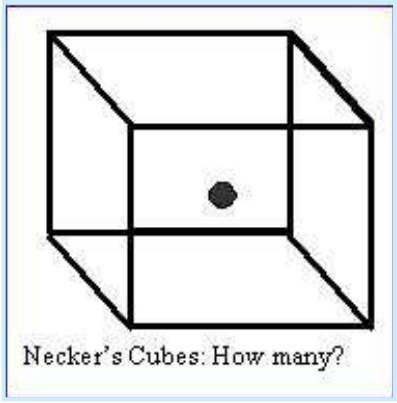
In this issue: Mark Fackrell from Melbourne , concludes his three part contribution by illustrating the potential benefits of general phase type models. And Chris Bain, responds from a clinician's point of view to Mark Mackay's (Adelaide) contribution in the last issue concerning "How can we get through the door. Thinking of that I recall the comment a reporter made to me about geriatrics n the 1970's *"It's just like pushing rice pudding through a sponge.*" We also have leads to several journal articles and a reminder for HSCMG Portrush 2008.

HSCM2008 Portrush 18th -20th March 2008
The Second International Health and Social Care Modelling Conference

Abstract submission for oral or poster presentation has been extended to December 15th.
Please submit your abstract by email to si.mcclean@ulster.ac.uk
Conference details topics and registration at:
http://info200.infc.ulst.ac.uk/events/hscm2008/organisation.html

Nosokinetics News: Where to next? Feedback needed.

In February 2008, Nosokinetics News should leave the nursery and go to school. This is our 24th edition. How should we proceed? What more should we do? What more could we do?

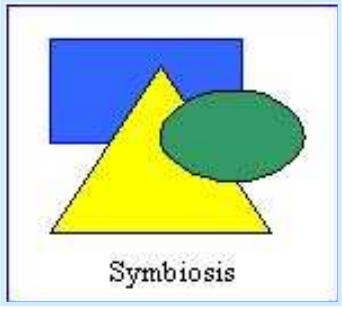


There are two sides to every question. Look at the Necker's cube. .How many cubes do you see? Last week I was discussing medical ethics with a group of eight MSc students. At the start only one saw two cubes.

There are two sides to every question. Nothing is ever black and white: even black and white. Our newsletter has two archives. The printed version at www.nosokinetics.org.uk and, thanks to Roy Johnston at IMS we also have an online version with an indexed author and subject list. See http://www.iol.ie/~rjtechne/millard/names.htm

Should we continue this symbiotic relationship? Is the double act, worthwhile? How should we proceed? Comments and suggestions welcome. Please mail to editor@nosokinetics.org. We look forward to hearing your views.

Talking of symbiosis, on the 25th February the Open Section of the Royal Society of Medicine is holding an evening meeting 6.00 p.m. to 8.00 p.m.



Don't want to; can't afford it; future of old people's medical care in the NHS: covenant to contract.
Joe Harris General Secretary of the National Pensioners Convention,
Can't Afford it, Don't Want to: Government Policies for Pensioners Care
Dr Chooi Lee; Consultant Physician, Kingston Hospital
Means Testing and the Bio/Psycho/Social Model of NHS care
Prof Peter Millard
Health and Social Care Systems: Symbiosis or Parasitism

Arguing that a symbiotic relationship needs excellence at both ends of the spectrum of care 'Top Down' and 'Bottom Up'. Whereas, focusing on only acute aspects of care is parasitic to the detriment of both acute and chronic care.

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Phase-type distributions in Health Care Modelling III

Mark Fackrell

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Editor's comment: In this, the last of his three articles, Mark explains why more general phase-type (*PH*) models, because of their versatility, should sometimes replace Coxian models in the analysis of health care data. Here we summarise the argument—the full version is attached and on the web.

Consider the histogram of some length of stay data shown in Figure1. If an order 6 general *PH* distribution is fitted to the data using the *EM* (*Expectation-Maximization*) algorithm (see Asmussen et al Scandinavian Journal of Statistics 1996, 23, 419-441) the resultant representation is

$$\boldsymbol{\alpha} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \tag{1}$$

$$\boldsymbol{T} = \begin{pmatrix} -3.2115 & 3.2115 & 0 & 0 & 0 & 0 \\ 0 & -3.2115 & 0 & 3.2115 & 0 & 0 \\ 0.6086 & 0 & -0.6272 & 0 & 0.0186 & 0 \\ 0 & 0 & 0 & -3.2115 & 0 & 3.2115 \\ 0 & 0 & 0.8053 & 0 & -0.8053 & 0 \\ 0 & 0 & 0 & 0 & 1.6479 & -3.2115 \end{pmatrix}. \tag{2}$$

This *PH* distribution cannot be a Coxian distribution (of any order) because some of the eigenvalues of ***T*** are complex numbers. The corresponding density function is also shown in Figure 1, and the fit is quite good the loglikelihood being — 11706.9226. Using the EM algorithm, an order 25 Coxian distribution is needed to achieve a fit with a greater log likelihood. Note also that the representation (**α,*T*)** has only 5 free parameters, with values 3.2115, 0.6086, 0.6272, 0.8053, and 1.6749. General order 6 *PH* distributions require 11 parameters, and here we have only five. This curious observation, which occurs quite often is not well understood. (See full article for references)

Mark continues his article by describing the fitting of *PH* distributions to model the pathways that patients take through hospitals. Figure 2 shows a schematic and simplified diagram. And to model the length of stay in this situation, general *PH* distributions would be a better choice that Coxian distributions.

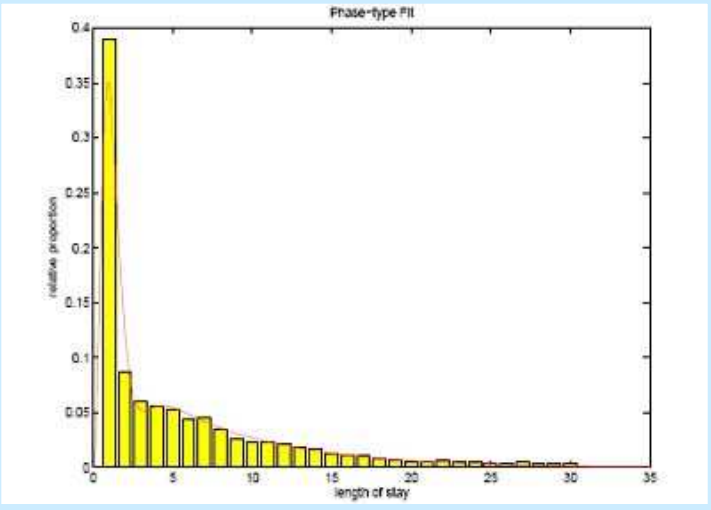

Figure 1. Order 6 PH fit to the length of stay

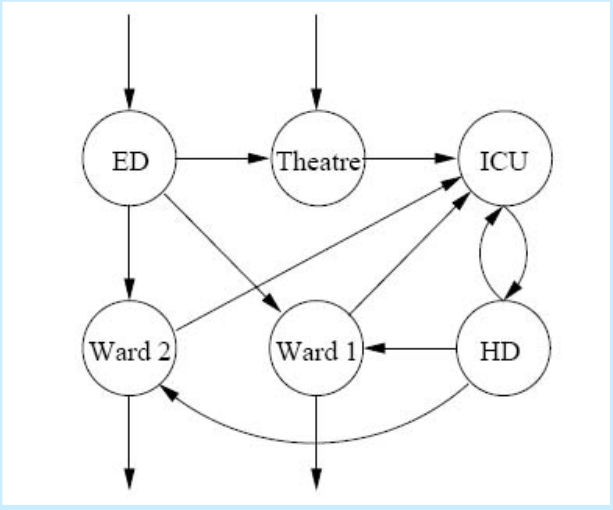

Figure 2. Schematic diagram for patient flow in a hospital.

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"Looking around from the inside ….."

Dr Christopher Bain MBBS, Master Info. Tech., MACS (Prov)

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Information Manager, Western and Central Melbourne Integrated Cancer Service, Melbourne

Editor's comment — Here Chris takes time out from looking after his young son (10 weeks) to respond from a clinicians point of view to Mark's article in the October issue. Chris describes the many internal barriers that need to be overcome before the Nosokinetics message gets through. Luckily, he keeps the smile on his face and remains committed to the cause. As sustainable, available, medical services for all citizens—young and old—is a goal worth striving for.

Background

As I have hit my 20th year of either being a clinician or working closely with them in the Victorian healthcare industry, I thought it might be useful to share some experiences from the front lines in response to Mark Mackay's article "Looking in from the outside - time to get through the door". My last two posts, including my current one, have been in supporting change in healthcare management and service delivery.....I must say it's a grim picture.

I should contextualise my comments by saying that I remain committed to the cause!! I believe we need much better ways to assist healthcare managers in many dimensions, and I see computer applications based on scientifically proven techniques and methodologies as being a key part of that. In addition, I continue to work in the area of "nosokinetics", for example I am working currently on a web based simulation project with the Victorian Partnership for Advanced Computing (VPAC http://www.vpac.org/) in conjunction with Dr Gitesh Raikundalia (http://suzuki.vu.edu.au/csm/staff.php?person=32) of Victoria University here in Melbourne. Management information systems for healthcare of various kinds remain my key career interest. I will focus my attention on hospitals in particular as they are the entities I have most experience with, and also where arguably, the biggest "bang for buck" is in relation to patient flow modelling activities.

The Environment – Is the Soil Culturally and Politically Barren?

In terms of the view from the "inside", I think we have a very difficult task ahead of us for a number of reasons, in addition to some of those that Mark has correctly identified from the "outside".

Tribes and sub-tribes certainly exist (Mackay, 2007; Fitzgerald & Teal 2003), but the problem goes deeper than that. I would argue that the number one problem is the effect of hospital culture and politics at all levels. In this tight funding environment which often sets up perverse competitive incentives (Drife and Johnston 1995) both in an operational and a research sense, managers (or even clinician-managers) can be driven towards personal survival and local performance optimization as key performance indicators (KPI's). To complicate matters even further, various stakeholders (eg clinicians, clinician managers, managers and patients) can even have very different perspectives on what are important facets or measures of hospital performance (for example, see the work by Tregunno, Baker et al. 2004)

The pressure that managers or clinician managers are under (Caplan 1994) does not easily allow them to take on the kinds of thinking necessary to give these technologies a chance. In one respect, who can blame them, in what is an incredibly cut throat environment. This environment is, sadly in my opinion, quite at odds with the nature of the work the industry is meant to perform. Unfortunately, anyone who has worked in a hospital system at a senior level for any length of time will be able to provide countless tales of the poor professional behaviours and undermining of productive activities that go on.

By way of illustration consider the work by King and McInerney (2006) from South Africa. They described factors affecting the resignations of registered nurses in an urban institutional setting. They found that "The resignations of registered nurses related to their physical working conditions and environment and included the following: unsupportive management structures, autocratic and dehumanizing management styles, negative stereotypy of nurses and the nursing profession, lack of autonomy in the workplace, professional jealousies and fractures within the profession inaccurate systems of performance assessment compounded by favouritism and racism……".

It could be argued that this represents an extreme description of hospital culture, unique to that country. Work form around the world, however, paints a less than flattering, although perhaps not as dramatic, picture of hospital culture and politics. For example, some of these findings are supported by a survey of 9638 nurses in Belgium by Milisen, Abraham et al. 2006. Norwegian research has identified (Skogsaas & Svendsen 2006) in interviews with hospital division leaders in relation to achieving change in a hospital setting – that "the most difficult challenge was to handle interactions dominated by suspicion, negative interpretation, assumptions and hidden agendas. Such interplays were the most limiting factor in the development of a common understanding of demands, goals and commitment to change processes across departments and units." Recent Canadian research (Cooper, Joglekar et al. 2005) highlights that even in areas of core hospital functioning, such as in critical care, there can be less than ideal communication processes. In particular it highlights how the publicity and appeals elements of a recognized "fairness" or "reasonableness" framework around decision making, is prone to breakdown in relation to decisions about ICU bed access. (see Table 1)

Table 1 : The four conditions of accountability for reasonableness	
Publicity	The decisions and reasons behind priority-setting decisions must be publicly available.
Relevance	These rationales must rest on evidence, reasons, and principles that fair-minded people can agree are relevant to deciding how to meet the diverse needs of patients in the context of limited resources.
Appeals	There is a process for revision and dispute resolution regarding priority-setting decisions.
Enforcement	There is a method of regulation in place to ensure that the first three criteria are met.
From Cooper *et al. BMC Health Services Research* 2005 5:67 doi:10.1186/1472-6963-5-67	

The existence of potentially destructive effects of rumours in healthcare organizations is also acknowledged (Robertson 2005) – not surprisingly as rumours are part of human nature. McCallin (2005) even raises the worrying notion that healthcare professionals "may need to learn how to collaborate". Unfortunately this observation also ties with practical experience on the ground. Feedback from the field in the UK (Freeman & Walshe 2004) supports the contention of poor progress in encouraging collaboration within and across health services-in the area of clinical governance in this particular case.

Furthermore, there is evidence of the destructive effect of poor relationships between clinicians and managers (Patterson & Bishop 2003). Atun (2003) also commented about this issue in the NHS, referring to the "doctor-manager divide and an unhealthy 'them and us' culture." He also stated at that time "Unsystematic efforts to bridge this divide have had limited success." I do not believe things have radically altered in the Victorian context since that time.

The evidence described above points to some of the manifestations of a less than optimal culture, and politics in healthcare, and in hospitals in particular.

Change and Achieving it

Another key hurdle I can identify is resistance to change. Mark has touched on this in his article but it is an enormous problem of its own. Grol and Wensing (2004) have identified that there are barriers to change in healthcare, even when there are well evidenced practice improvements to be made. Cabana, Rand et al. (1999) identify "habit" and "routines" as reason why clinicians don't follow clinical practice guidelines. Others have noted (Robinson & Turnbull 2004) that organizations need to foster practice change if recommendations are to be taken up and made sustainable - that is to say organizational culture (Skogsaas & Svendsen 2006; Seren & Baykal 2007) also has a role in aiding or obstructing change, even at the basic clinical care level, let alone with something potentially seen as much less immediate to day to day hospital functioning such as patient flow modelling.

There are also many reports in the medical IT/informatics literature that highlight clinician resistance as a reason for failure in IT projects in health (it's not a long bow to draw the analogy with the kind of work we are trying to foster under the banner of "nosokinetics"). From my on the ground experience, currently working in Cancer service reform across six hospitals, this phenomenon is alive and well.

My personal musings on this include an observation that medicine is a very traditional and conservative discipline by its' nature and people who remain, to their credit, clinicians for long periods of time (and who therefore often rise to management positions), are by their nature unlikely to welcome change (Roig JV, Rodríguez-

Carrillo R et al. 2007). In addition, as I have alluded to previously, in some cases they are even politically motivated to shoot it down.

How to "get through the door"?

How we as a "community of practice" ("a group of people informally bound together by shared expertise and passion for a joint enterprise (Wenger and Snyder 2000)", can address these barriers is not an easy question to answer unfortunately. We'd all be bowled over in the rush to take up our various discoveries and innovations if it were.

Again, as Mark points out, the need is there (Green & Nguyen 2001) and has been for some time (Pasley, Lagoe et al. 1995). I think Mark is spot on with his point about answering the questions people are grappling with, and the potential benefits of so doing in his comments about leaving them (the "customers") with something (Mackay 2007). In my opinion, the gap between academic endeavour and the hard nosed world I have briefly outlined above is too great for a traditional academic world view to penetrate on a consistent and useful basis. These "customers" need "simple", practical solutions. (for simple – read "where all the inevitable complexity is well hidden from them, but available for them to access if they wish to discuss it and understand it"). Small quick wins are also seen as a way to build trust and engender interest in relation to change in health care (Page 2003; Boxall & Flitcroft 2007) as Mark has also mentioned (Mackay 2007).

Conclusion

In conclusion - I write this outline of the current state of the barriers we face as a spotlight on some very real issues that may be more apparent from the "inside" than the "outside", as we all try to advance the cause of "evidence based management" for want of a better term. In order to succeed we will all need to continue to collaborate on solutions in a fashion cognoscente of these harsh realities.

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Roig JV, Rodríguez-Carrillo R, et al. (2007). "Multimodal rehabilitation in colorectal surgery. On resistance to change in surgery and the demands of society." Cir Esp. **81**(6): 307-15.

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Focusing on cost of care

COPD: Chronic Care Model reduces cost and LOS in COPD
Adams, S. G., P. K. Smith, et Al. "Systematic review of the chronic care model in chronic obstructive pulmonary disease prevention and management." Archives of Internal Medicine 167, no. 6(2007): 551-61
A 2005 literature review of found no significant differences in symptoms, quality of life, lung function, and functional status between the intervention and control groups. However, the group with two or more interventions had fewer emergency/unscheduled visits and hospitalizations and a shorter length of stay compared with control groups.

A&E (ED) Severity of illness increases cost

Dong, S. L., M. J. Bullard, et Al "Predictive validity of a computerized emergency triage tool." Academic Emergency Medicine 14, no. 1(2007): 16-
eTRIAGE, a web-based decision support tool, is based on the Canadian Triage and Acuity Scale (CTAS), a five level triage system (CTAS 1 = resuscitation, CTAS 5 = nonurgent). During a six month period, 29,524 patients were triaged. The odds ratio ($p<0.001$) for admission, and costs were greater in CATS 1 and 2 compared with 4 and 5. and CATS 1 were more likely to die, which given the nature of illness is not unexpected.

Cardiac surgery Adverse events increase cost

Ehsani, J. P., S. J. Duckett, and T. Jackson. "The incidence and cost of cardiac surgery adverse events in Australian (Victorian) hospitals 2003-2004." Eur J Health Econ
Linear regression modelling, adjusted for age and co-morbidity was used. 36.8% of 16,766 multi-day cardiac disease cases had at least one adverse event. They stayed approximately 7 days longer and had four times the case fatality rate. The total cost of adverse events for each DRG was AUS$42.8 million, representing 21.6% of total expenditure on cardiac surgery and adding 27.5% in broad terms to the cardiac surgery budget.

Academic Teams decrease cost of care

Everett, G., N. Uddin, and B. Rudloff. "Comparison of hospital costs and length of stay for community internists, hospitalists, and academicians." Journal of General Internal Medicine 22, no. 5(2007): 662-
This single institution retrospective cohort study reports that academic Internist teams had lower cost and LOS compared to traditional private Internists (30% and 40%) and private Internists (24% and 30%). Hospital mortality was equivalent for all groups. Academic teams had 2.3-2.6% more 30-day readmissions than the other groups.

Life style influences outcome

Burke, V., Y. Zhao, et Al. "Health-related behaviours as predictors of mortality and morbidity in Australian Aborigines." Preventive Medicine 44, no. 2(2007): 135- A 1988-89, survey of Western Australian Aborigines (256 women, 258 men) aged 15-88 years documented their diet, alcohol and smoking habits. A longitudinal study using 2002 data and Cox regression found clustering of adverse behaviours is common and increases the risk of coronary heart disease and death.

Modelling Pathways

Adeyemi, S., T. Chaussalet, H. Xie, and P. Millard (2007) Patients Flow: A Mixed-Effects Modelling Approach to Predicting Discharge Probabilities, pp. 725-730
A class of generalized linear mixed models is used to capture individual patients experience during the process of care as represented by their pathways through the system. The approach could predict the probability of discharge from the system, as well as detect where the system may be going wrong.

A&E Unintended consequences

Han, J. H., C. Zhou,et Al. "The effect of emergency department expansion on emergency department overcrowding." Academic Emergency Medicine 14, no. 4(2007): 338-43
An increase in ED bed capacity did not affect ambulance diversion. Instead, total and admission hold LOS increased. As a result, ED expansion appears to be an insufficient solution to improve diversion without addressing other bottlenecks in the hospital.

Second International Health and Social Care Modelling Conference (HSCM 2008)

Portrush, Northern Ireland 18 - 20 March, 2008

http://info200.infc.ulst.ac.uk/events/hscm2008/themes.html

Organised in association with the University of Ulster, School of Computing Information and Engineering at the Coleraine Campus, HSCM 2008 enables researchers and practitioners to meet in a convivial setting to, exchange ideas, examine the current modelling trends and issues, and develop new solutions and research directions to ultimately, improve patient and client care.

The conference fee includes two nights accommodation, and full board at the Comfort Hotel at the Ramada hotel in Portrush http://www.comforthotelportrush.com , a small seaside town on the North Coast of Ireland, with beautiful beaches, convivial restaurants and friendly pubs. It is close to the Bushmills Distillery and Giant's Causeway and part of the Causeway Coast Area of Outstanding Natural Beauty.

Abstract Submission: 15th December 2007

Abstracts of one A4 page are invited for oral or poster presentation. Format: Font Times New Roman 12 point, 1.5 spacing, single column, margins: left 3cm; right 2.5cm; top & bottom 3cm; file type MS Word (preferred). Please submit your abstract by email to Sally McClean at si.mcclean@ulster.ac.uk

For further details contact Sally McClean (si.mcclean@ulster.ac.uk).

Clinical and Practical
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Keynote Speakers: *Professor Thierry Chaussalet (University of Westminster), Professor Don Campbell (Monash University), Dr Ken Fullerton (Queens' University Belfast), Professor Gary Harrison (College of Charleston), Professor Peter Millard (St. George's Hospital Medical School), and Professor Terry Young (Brunel University).*

Nosokinetics News is the newsletter of the UK Nosokinetics Group

Nosokinetics is the science / subject of measuring and modelling the dynamic aspects of patient and client movement (flow) through health and social care systems. From the Greek, literally, *noso* (sickness) and *kinetics* (movement).

The group collaborates to organise conferences and disseminates news of our and others research and practical use of modelling to enhance decision making in health and social care systems.

Thanks to IMS our web archive of full texts of submitted papers is at: http://www.iol.ie/~rjtechne/millard/index0.htm

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Phase-type Distributions in Healthcare Modelling III

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In the last of these three articles on phase-type (PH) distributions in healthcare modelling, I will suggest that more general PH distributions be considered when modelling systems. As explained in the previous article, Coxian distributions have been used predominately in healthcare modelling, mainly because of their simplicity and ability to give some sort of interpretation to the systems being modelled. However, more general PH distributions may sometimes be more useful because of their greater versatility.

Consider the histogram of some length of stay data shown in Figure 1. If an order 6 general PH distribution is fitted to the data using the EM (*Expectation-Maximization*) algorithm (see Asmussen, Nerman, and Olsson [1]) the resultant representation is

$$\boldsymbol{\alpha} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \tag{1}$$

$$\boldsymbol{T} = \begin{pmatrix} -3.2115 & 3.2115 & 0 & 0 & 0 & 0 \\ 0 & -3.2115 & 0 & 3.2115 & 0 & 0 \\ 0.6086 & 0 & -0.6272 & 0 & 0.0186 & 0 \\ 0 & 0 & 0 & -3.2115 & 0 & 3.2115 \\ 0 & 0 & 0.8053 & 0 & -0.8053 & 0 \\ 0 & 0 & 0 & 0 & 1.6479 & -3.2115 \end{pmatrix}. \tag{2}$$

This PH distribution cannot be a Coxian distribution (of any order) because some of the eigenvalues of $\boldsymbol{T}$ are complex numbers. The corresponding density function is also shown in Figure 1, and as we can see, the fit is quite good - the loglikelihood being -11706.9226. Using the EM algorithm, an order 25 Coxian distribution is needed to achieve a fit with a greater loglikelihood. Here, it appears, using a general PH representation is superior to using a Coxian representation. Indeed, if the representation (1)–(2) is needed in the calculation of performance measures, the smaller representation will be much easier to compute with than a larger Coxian representation. Note also that the representation $(\boldsymbol{\alpha}, \boldsymbol{T})$ has only 5 "free" parameters, with values $3.2115, 0.6086, 0.6272, 0.8053$, and 1.6479. Recall that a general order 6 PH distribution requires 11 parameters, and with this particular example we have observed an even further reduction in the number of parameters needed. This curious observation occurs quite a lot when fitting PH distributions to data, but is not well understood. For some discussion on this aspect of PH fitting see Faddy [3] and [4], and Hampel [5].

Figure 2 shows a schematic and simplified diagram for patient flow in a hospital. Patients enter the hospital via the emergency department (ED) (state 1), or as elective patients requiring surgery in the theatre (state 2). After spending time in the ED patients can move to the theatre, or to one of the two wards (states 5 and 6). From the theatre patients go to the intensive care unit (ICU) (state 3), and then on to the high dependency

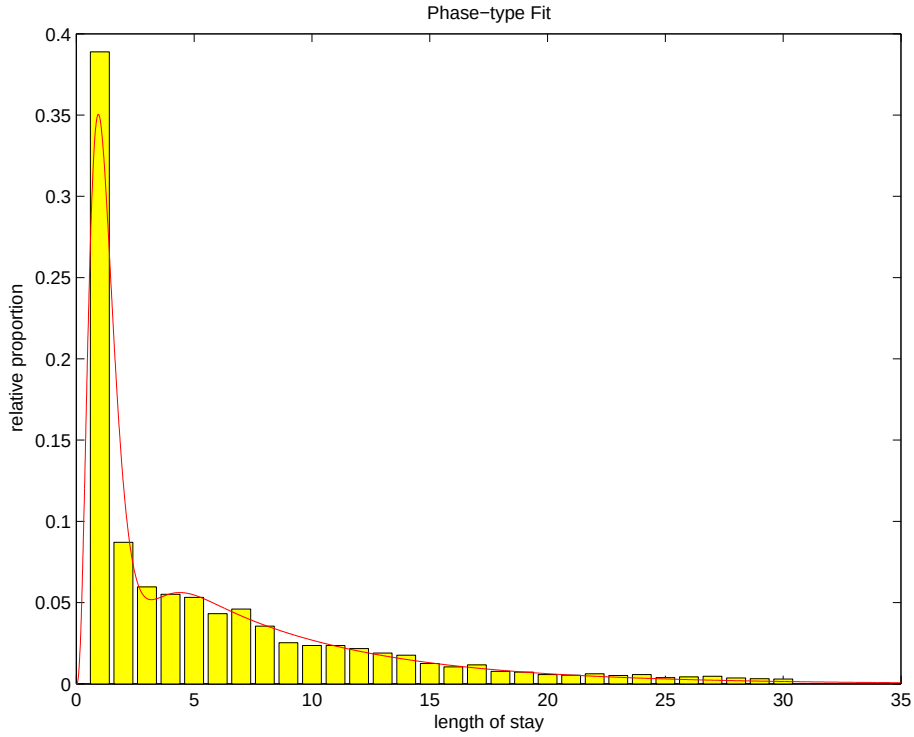

Figure 1: Order 6 *PH* fit to the length of stay histogram.

ward (HD) (state 4), before moving on to one of the two wards. At any time, patients may need to be readmitted to the ICU from HD or a ward, or they may exit the wards by being discharged or dying.

If we need to model the length of time a patient stays in hospital, we could model the length of stay in each unit with a *PH* distribution, and then combine them according to the structure shown in Figure 2 to form a larger *PH* distribution. For, $i = 1, 2, \ldots, 6$, if unit i is modelled with an order p_i *PH* distribution, such a *PH* distribution would have a representation

$$\boldsymbol{\alpha} = \begin{pmatrix} \boldsymbol{\alpha}_1 & \boldsymbol{\alpha}_2 & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \end{pmatrix} \tag{3}$$

$$\boldsymbol{T} = \begin{pmatrix} \boldsymbol{T}_{11} & \boldsymbol{T}_{12} & \mathbf{0} & \mathbf{0} & \boldsymbol{T}_{15} & \boldsymbol{T}_{16} \\ \mathbf{0} & \boldsymbol{T}_{22} & \boldsymbol{T}_{23} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \boldsymbol{T}_{33} & \boldsymbol{T}_{34} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \boldsymbol{T}_{43} & \boldsymbol{T}_{44} & \boldsymbol{T}_{45} & \boldsymbol{T}_{46} \\ \mathbf{0} & \mathbf{0} & \boldsymbol{T}_{53} & \mathbf{0} & \boldsymbol{T}_{55} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \boldsymbol{T}_{63} & \mathbf{0} & \mathbf{0} & \boldsymbol{T}_{66} \end{pmatrix}. \tag{4}$$

Here, $\boldsymbol{\alpha_1}$ and $\boldsymbol{\alpha}_2$ will be nonnegative and nonzero vectors of lengths p_1 and p_2, respectively, and $\boldsymbol{T}_{ii}$ is an order p_i *PH* generator. The nonzero off-diagonal matrices are all nonnegative, and have size $p_i \times p_j$ whenever $(i, j) \in \{(1,2), (1,5), (1,6), (2,3), (3,4), (4,3), (4,5), (4,6), (5,3), (6,3)\}$.

The simplest way to fit such a *PH* distribution to data would be to assume that the time spent in each unit is exponentially distributed, and then use the *EM* algorithm to fit an order 6 PH distribution of structure (3)–(4). Alternatively, an exponential distribution could be fitted to the length of stay data for each unit individually, and when the

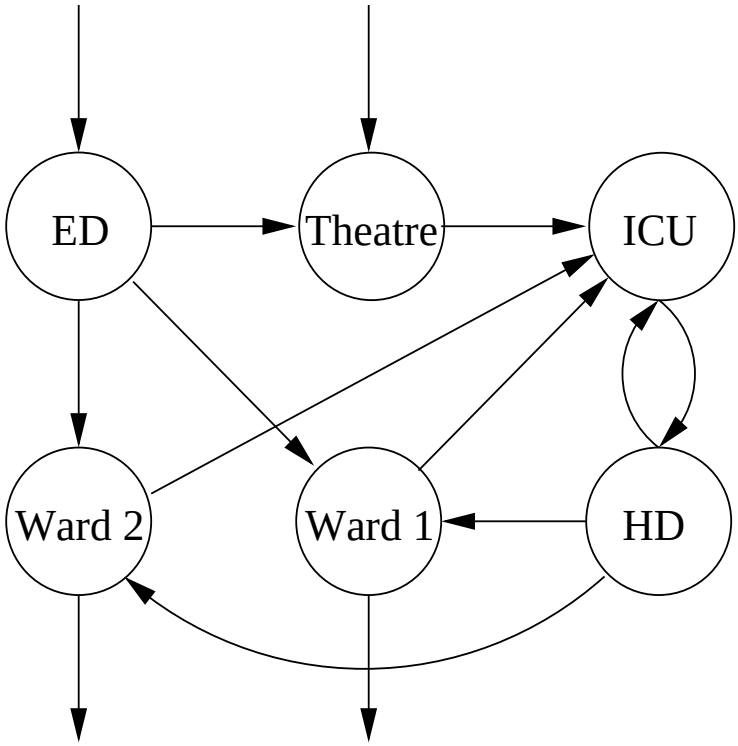

Figure 2: Schematic diagram for patient flow in a hospital.

proportions of patients moving between the units is estimated, the *PH* distribution could be constructed. This approach could be more accurate but the times patients stay in each unit needs to be recorded, rather than the total length of stay as with the former approach.

A more sophisticated approach would be to model the length of stay in each unit with a higher order *PH* (or Coxian) distribution. But here the overall representation would be quite large and the computation time taken for a good fit could be long. Also, if the time spent in each unit was fitted with a *PH* distribution individually, there is no straightforward way in which to estimate the nonzero off-diagonal matrices.

Nevertheless, to model the length of stay in this situation, general *PH* distributions would be a better choice than Coxian distributions.

A more detailed account of the use of *PH* distributions in the healthcare industry, and a comprehensive bibliography can be found in Fackrell [2].

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