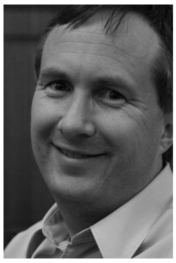
Issue 4.5 October 2007

Nosokinetics News

Bi-monthly Newsletter of the UK Nosokinetics Group

In this issue:

Two Mark's, both Australian — Mark Fackrell, Melbourne, continues his phase-type tutorials; Mark Mackay, Adelaide, considers "How can we open the door?". From Thierry Chaussalet's group, *FLoSC*, a tool to forecast length of stay and costs in social care is described. Shola Adeymi, PhD student, reveals some mathematical mysteries in modelling pathways. I muse about medieval fish traps, bed-blocking and the tide coming in. Ludwig Kuntz's research into the process of care in German hospitals. And we give details of abstract submission and registration for HSCM2008 being organised Sally McClean at the University of Ulster.



Congratulations to Mark Mackay

His University of Adelaide, PhD thesis on "Compartmental Modelling of Acute Care Hospital Bed Occupancy for Strategic Decision Making" has passed the exam, with minor alterations. Once those are done we can then, happily address him as Dr. Mark Mackay, and put the abstract into the newsletter. Mark's original contribution is to demonstrate, using data from Australia and New Zealand, the applications of compartmental flow modelling to forecasting in long term and short term outcomes of change.

Phase-type Distributions in Healthcare Modelling 2

Mark Fackrell, *Dept. of Mathematics and Statistics, University of Melbourne, Australia*

The second of Marks' three articles on phase-type (*PH)* distributions, attached to this newsletter, describes Coxian distributions. These special types of *PH* distributions are particularly useful in health care modelling.

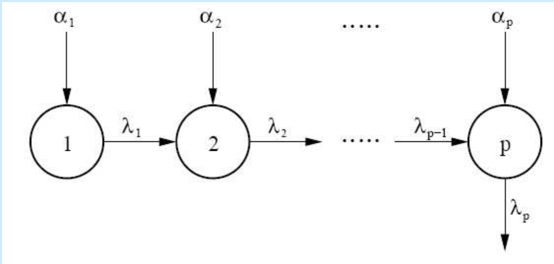


Figure 1: State transition diagram for an order p Coxian distribution.

Figure 1 shows the state transition diagram for an order p Coxian distribution. Notice that the representation (1)-(2) depends on only $2p$-1 parameters, whereas General *PH* representations require $p^2 + p - 1$ parameters. Thus general *PH* distributions are over parameterized, whereas Coxian distributions are not.

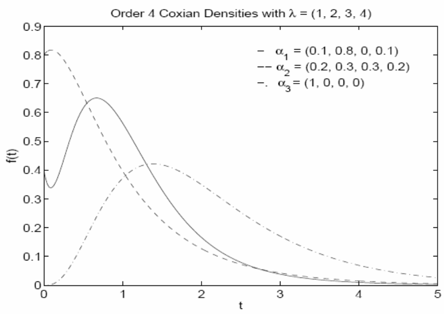
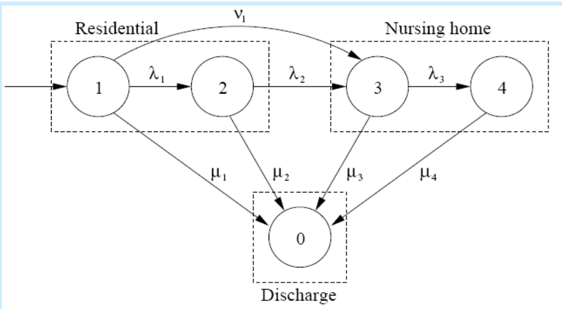


Figure 2: Density functions for three different order 4 Coxian distributions with $\lambda_1 = 1$, $\lambda_2 = 2$, $\lambda_3 = 3$, $\lambda_4 = 4$.

The shapes in Figure 2, from a three order 4 Coxian distribution with the same generator, but different vectors, exhibit much more flexibility than an exponential function, which should come as no surprise as they rely on 7 free parameters instead as just one! Indeed Coxian distributions are more versatile than both hyperexponetial distributions and generalized Erlang distributions, which also depend on $2p$-1 parameters.

Mark illustrates the use of Coxian distributions with the state transition model used by researchers from the HSCMG at the University of Westminster. By coincidence, serendipity, no editor's tricks were involved.



Also see *FLoSC* on Page 3 of this Newsletter showing the practical potential of this modelling tool.

The full text of Mark's paper is in the IMS archive at (see below) and in the Tutorial section of the Nosokinetics website at http://www.nosokinetics.org.uk/.

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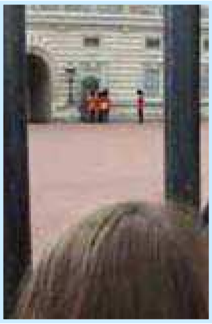
Looking in from the outside – time to get through the door

Mark Mackay, *Principal Project Officer, Department of Health, South Australia*



Looking in at Buckingham palace

Editor's comment: Medicine used to be an Art based Science, now it's a Science based Art. Mathematical modelling is not a forest fire. Indeed its opaque, but as Haifeng Xie said one day, "Maths is boring, but there are exciting things you can do with it". Given, for every equation one has in a paper", Sally McClean says "You loose 50% of the readers". There is a brave new world, shorn of constant change, but "How do you get the message across?" Also see MASH News in the 2006 August (3.4) Nosokinetics News.

Staking our place Previously I have suggested that it is time to stake a place in the "health care problem" ground by health care modellers, particularly in relation to those modellers involved in patient flow modelling activities (see Mackay, 2006). In this NK article I outlined that the health care sector is perhaps different to other industries – there are many players involved in decision-making and no-one group is likely to be able to solve the current problems being experienced across health systems without involvement of members from some of the other players.

Tribes and sub-tribes The players include doctors (and don't ignore their sub-tribes - the specialties and sub-specialties), nurses, administrators, bureaucrats, politicians, academics, consultants, and somewhere among these tribes is the small group (or individual) of modellers. Having worked in the health sector for some time it is interesting to observe how the tribes, and even more so the sub-tribes, can come together on some matters and yet be at war on other matters. Thus, working the system can be complex.

Cottage industries Furthermore, while "hospitals" are often viewed akin to the large factory, in reality it is a large building occupied by various cottage industries that co-exist and share facilities and services as required. A large bit of the glue binding the services is the hospitality services required by patients. Clinical and diagnostic services are also part of the glue (e.g., theatres and imaging services). Furthermore, the variation in practice and problem is far greater than any widget producing factory that I've seen – and I've seen a few as a consequence of employment in other sectors other than health.

Complexity So it's a complex environment, what's the big deal I hear you ask? Modelling has the potential to help improve decision-making in the health care sector, particular in relation to patient flow issues. Yet modelling doesn't appear to have "cracked" the potential that exists in the health sector. Just as the sector itself is complex, the reason for not cracking the market is undoubtedly also complex.

Getting heard Currently I think that health care modellers often sit on the outside and are yet to get heard at the main table. Reasons for not getting heard include the variety of modelling tools that exist and aren't well understood by the non-modelling tribes (language and tool barriers); the results are sometimes at best variable (see Fone, Hollinghurst et al. 2003); sometimes there's not enough resources to enable inclusion of the modellers; the solutions provided by modellers are not sufficiently timely (i.e., can't meet delivery deadline of answering the problem yesterday in a political and reactive environment); and (or) don't solve the problem (either in a desired way or identify things that people really don't want to hear).

So what do I do? I could go on. Or I could give up.

The fountain of Trevi in Rome

Right time Given the pressures beginning to occur on hospitals as the volume of baby boomers with health problems increases and the forecast consequences of chronic diseases (not affecting just the older members of the community, but increasingly younger members as well), the time is probably most conducive for getting through the door and being heard at the table.

Tipping Point But how can the "tipping point" (Gladwell, 2000) be reached? While I suggested some mechanisms in the NK article last year, many of these are things that will have an impact over time, such as education. However, we need to be going through the door now or at least sometime very soon. (*continued on page 3*)

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Plain language The book adaptive business technology (Michalewicz et al., 2006) is perhaps a good starting point. It's in plain language. While the concepts it deals with are undoubtedly complex, it provides a great example of how to communicate the gains decision-makers can make from the application of data-mining and modelling. Plain English (or other language if not in an English speaking country) is a key – it provides a means of reaching across the divide so that others can understand what is being offered.

Answer the questions people are grappling with. This provides a real tension for academics – the priority is often to publish and the application may or may not be a priority. It may also have to be at a level that is probably less complex than some academics might like. But without opening the door, there is no entrée to showing those already at the table the vast arrange of value that can be added by "the modeller".

Leave them with something – some basic tool that is based in software that they probably are comfortable in using (even if only at a basic level) and don't charge them for it. They may never use it, but they didn't pay for it (unless of course specified as part of the work). It provides the opportunity to exchange and build up trust. This will give us "the champion" and "the champion" will help us to reach the tipping point.

The setting sun heralds a new dawn.

"Some business model?" you say. Well, if it's good, they'll come back and you'll get more papers or more consulting (or whatever it is that is driving you to be in this game). And the end result… may be the modeller will be contributing more to the future of the health sector, particularly in relation to patient flow issues.

References

Mackay M (2006). Nosokinetics – staking our claim. *Nosokinetics News*, June, Vol 3.3 pg 4-5.
Gladwell M (2000). *The Tipping Point: How Little Things Can Make a Big Difference.* (ISBN 0-316-31696-2) first published by Little Brown and Company, Great Britain.
Michalewicz Z, Schmidt M, Michalewicz M and Chiriac C (2006). *Adaptive Business Intelligence*. Springer.
Fone D, Hollinghurst S, et al. (2003). Systematic review of the use and value of computer simulation modelling in population health and health care delivery, *Journal of Public Health Medicine*, 25(4): 325-335.

Academic research to practical use: A step forward by the HSCMG

FLoSC

Forecasting Length of Stay and Cost

Research by the Health and Social Care Modelling Group (HSCMG) at the University of Westminster, in collaboration with the London Borough of Merton, has led to the development of a practical decision support tool *FLoSC*, which is a toolkit helping Local Authorities to analyse the patterns of length of stay of their publicly funded clients in institutional long-term care.

FLoSC forecasts the cost of existing, known commitments i.e. how much money will be needed this year and in future years for the group of publicly funded residents currently in residential and nursing home care. So, using their data, local authorities can estimate the money available for new admissions and determine how much of next year's budget is available for home and institutional care. The toolkit can also be used to compare forecasts under different cost scenarios, which may reflect possible future changes in pricing or costing policy. These functionalities are of crucial importance to the planning of a successful budget for long-term care at local authority level.

The development of *FLoSC* was funded by the Care Services Efficiency Delivery (http://www.csed.csip.org.uk/) program of the Department of Health, UK.

For upcoming information and updates about *FLoSC*, please visit http://www.healthcareinformatics.org.uk/ or email Thierry Chaussalet (chausst *at* wmin *dot* ac *dot* uk).

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Patient flow: A mixed-effects modelling approach

Shola Adeyemi, *PhD student, HSCMG, University of Westminster, UK*

Introduction

In the studies of patient flow and LOS modelling, an individual patient's experience during the delivery of care is often overlooked. In order to have an insight into what an individual patient might experience during the course of care, we propose a modelling framework for patient pathways through the healthcare system based on mixed effect models.

Modelling approach

Health care delivery can be seen as a multi-state system; a patient's experience in the system is captured by the different states (locations or health states) visited by the patient. This is also known as *patient pathway*, which essentially is a collection of clustered or repeated observations of a patient. Mixed effect models, which incorporate fixed effects (e.g. the systematic component capturing the population correlation structure) and random effects (e.g. the random component capturing patient specific effects) are naturally suited in such a modelling situation.

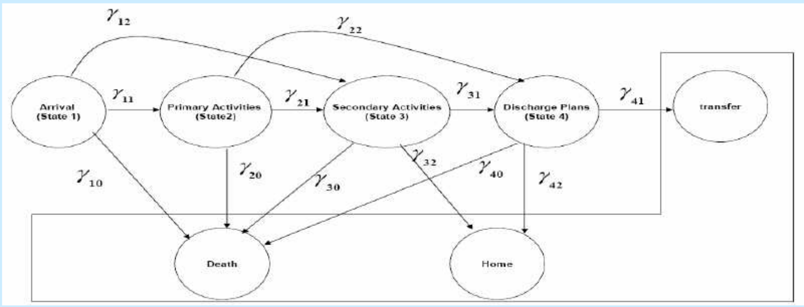

Figure 1: An artificial healthcare system.

We illustrate this modelling concept using an artificial healthcare system depicted in Figure 1. We assume that patients entering the system will undergo some interventions and then progress from doing primary, secondary and discharge planning activities before discharge, which includes discharge by death, discharge to patients' home or transfer to another healthcare institution. In a (non-absorbing) state, we model the probability of going through any path (e.g. an outgoing arrow) using a multinomial formulation. Furthermore, we introduce the concept of health status of a patient, which is modelled as a random effect. The idea is that health status of a patient is the underlying driving force of movements in the system. Health status can be further modelled by a set of covariates if such data is available.

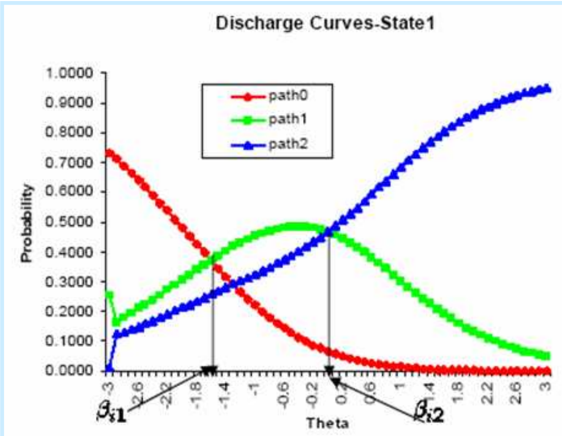

Figure 2: Discharge probability along different paths.

For illustration, Figure 2 show the effects of changes in the health status of a patient on the probability of discharge in state 1. Since there are three paths leaving the state, there are two threshold values (e.g. $\beta_{\iota 1}$ and $\beta_{\iota 2}$) for the health status value of a patient, above which the likely path will change. In this case, path 0 and path 1 intersect at $\beta_{\iota 1}$ = -1.5, and path 1 and path 2 intersect at $\beta_{\iota 2}$ = 0.1. These thresholds show that the most likely path is path 0 when a patient's health status measure is below $\beta_{\iota 1}$; and path 1 is the most likely between $\beta_{\iota 1}$ and $\beta_{\iota 2}$; while path 2 is the most likely above $\beta_{\iota 2}$. Note that if $\beta_{\iota 2} < \beta_{\iota 1}$, it suggests that path 1 will not be most likely at any point. This might be interpreted as indicating a problem with path 1. However, this is not always the case.

Conclusion

We have demonstrated that mixed effect models can play an important role in modelling patient pathway. Further work is currently underway to improve this modelling framework. For more detail of the model, please see:

Adeyemi, S., Chaussalet, T., Xie, H. and Millard, P. (2007) Patients flow: a mixed-effects modelling approach to predicting discharge probabilities. In: *Proceedings of the 20th IEEE International Symposium on Computer-Based Medical Systems*, pp. 725-730, IEEE CBMS 2007, Maribor, Slovenia, 20-22 June 2007.

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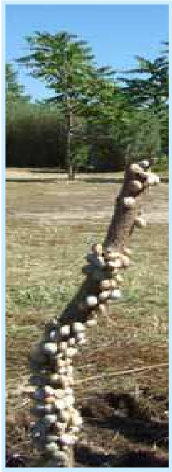
Global warning: high tide is on its way; old people are coming and they won't go away

Peter Millard

Campervanning on the Isle de Reé, on the Atlantic coast of France. Banned from internet cafes, no emails what bliss. Reading Preserved Smith's (1934) *History of Modern Culture, 1678-1776*, as one does, thinking about the enlightenment that followed Newton's laws. Looking at nature stacking sea shells on plants and exploring medieval fish traps, my thoughts turned to 'bed blocking' and the computational and conceptual errors that underpin the world wide crisis in hospital care.



Bed blocking

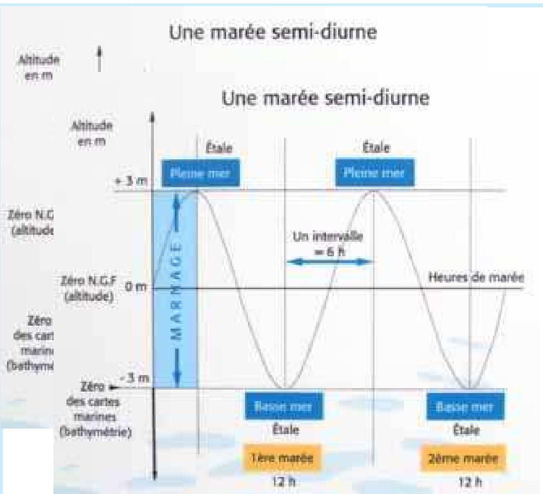
The Isle de Ré, is only 30 kilometres long and 5 kilometres wide with no hills. It's a cyclists paradise. It has no hills and separate cycle tracks. Moreover, there are bicycle hire shops in all the villages and, when tired, buses to bring you and the bike back to the base. Moreover, it has a fascinating history. Both as a military base protecting La Rochelle, and as a source of salt and fish.

"What's that to do with Nosokinetics?" you may well ask. Well in the middle ages the island had 141 fish traps, now only six remain. Built and repaired by hand, damaged frequently by the storms, both little and big fish are caught as they come in on the tide. Vertical Iron bars at the 'holes in the walls' ensure that big fish stay in and little fish are netted as they go out with the tide. So, that's a two compartment model if ever I saw one.

Poster illustrating a medieval fish trap and the shape of the Ile de Ré

Later we visited the museum of the sea and saw for the first time the mathematics of the tides. Analogies abound. Could the mathematics of the rising and falling tides be adapted to help solve the problem of seasonality in hospital care. High tides midday, low tides at night.

Everyone knows that rivers are more likely to break their banks and overflow at midnight. So defences are planned accordingly. Why then is hospital bed occupancy still measured in nights?



High tides and low tides. As the sea comes in and the sea goes out. In hospitals the bath is never empty, and patients come in with the tide.

Also days of the week and seasons of the year influence the need for acute hospital care? The modern emphasis is on fast track treatment and plans are being made accordingly. But, how can you plan for that if you don't know the hour at which in patients arrive and leave.

So errors abound. Like the tidal seas, hospital beds are never completely empty and different numbers and types of big and little fish come in with the tide.

Until we recognise the need to measure the process of patient care in minutes, hours, days, weeks, months and years, the medieval fist traps tell us we will never solve the problem of care. Also, like the fish traps, comprehensive hospital and community services take years to build and days to smash down.

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Modernising Acute Hospital Care

Editor's comment. If you are interested in the management theories that underpin the modernisation of acute hospitals, three recent papers from Ludwig Kuntz' group are well worth reading. They opened my eyes.

Incorporating efficiency in hospital-capacity planning in Germany

Kuntz, L., S. Scholtes, et al. (2007). European Journal of Health Economics **8**(3): 213-23.

Bed occupancy, as a measure of hospital efficiency, neglects important drivers of economic efficiency, such as treatment costs and case mix. An alternative metric is developed and used in the hospital-capacity planning cycle in a German federal state. 92 hospitals were involved. Standard hospital data, including annual costs, number of cases, disaggregated by medical departments, ICD codes, length-of-stay, certified beds, and occupancy rates were used. 18 hospitals were deemed to be inefficient and targets for over-proportional capacity cuts and 15 were identified as efficient hospitals. The developed model and analysis has affected the federal state's most recent medium term planning cycle.

Process-Based Organization Design and Hospital Performance

Vera, A. and L. Kuntz (2007). Health Care Management Review **32**(1): 55-65.

Lean thinking, continual quality assurance, six sigma are all forms of process based change. The economic aim being to drive down costs, while improving quality of product, i.e. satisfied customers buying the product. The research hypothesis was *"Hospitals that exhibit a high degree of process orientation in their organization design are more efficient than hospitals with a low degree of process orientation".* Using data development analysis, factor analysis and regression analysis and data gathered by questionnaire from 43 chief executives the researchers found had a high degree of process-based-reorganization has a moderate but significant positive effect on efficiency. With the caveat, that implementation rules, physician involvement and an adequate organizational culture are essential components of success.

Modular organization and hospital performance.

Kuntz, L. and A. Vera (2007). Health Services Management Research **20**: 48-58.

Modularization is a modern form of organization, which vertically disaggregates a firm and uses market mechanisms within hierarchies. This paper examines whether the use of modular structures has a positive effect on hospital performance. Using multiple regression analysis, the main result is that modularization does not have a positive effect on hospital performance. However positive efficiency effects of two central ideas of modularization, namely process orientation and internal market mechanisms were found..

MASHnet

HEALTH CARE MODELLING & SIMULATION

BUILDING ON EXPERIENCE

on WEDNESDAY, NOV 21st 2007
at UNIVERSITY OF WESTMINSTER, LONDON

This one-day workshop will focus on the practical aspects of integrating modelling and simulation solutions in healthcare. It will draw on examples of case study applications and the lessons learned. Delegates will be encouraged to participate through sharing of practical issues and experience. The workshop will appeal particularly to healthcare professionals (at all levels) with an interest in using these approaches to improve service provision, academics and researchers in this area, commercial representatives who use or develop tools for healthcare modelling. Time: 10.30 – 17.00.

The cost for the day will be £40 for bookings received before Nov 1st (£50 after that date). This will include refreshments during the day and full buffet lunch. Places are limited to 80 participants and early booking is advised.

(Email enquiries can be directed to: joanne.perry@pms.ac.uk)

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Second International Health and Social Care Modelling Conference (HSCM 2008)

Portrush, Northern Ireland 18 - 20 March, 2008

http://info200.infc.ulst.ac.uk/events/hscm2008/themes.html

Organised in association with the University of Ulster, School of Computing Information and Engineering at the Coleraine Campus, HSCM 2008 enables researchers and practitioners to meet in a convivial setting to, exchange ideas, examine the current modelling trends and issues, and develop new solutions and research directions to ultimately, improve patient and client care.

The conference fee includes two nights accommodation, and full board at the Comfort Hotel at the Ramada hotel in Portrush http://www.comforthotelportrush.com , a small seaside town on the North Coast of Ireland, with beautiful beaches, convivial restaurants and friendly pubs. It is close to the Bushmills Distillery and Giant's Causeway and part of the Causeway Coast Area of Outstanding Natural Beauty.

Abstract Submission: 30th October 2007

Abstracts of one A4 page are invited for oral or poster presentation. Format: Font Times New Roman 12 point, 1.5 spacing, single column, margins: left 3cm; right 2.5cm; top & bottom 3cm; file type MS Word (preferred). Please submit your abstract by email to Sally McClean at si.mcclean@ulster.ac.uk

Acceptance Notice: 23rd November 2007
Registration Early-Bird Deadline: 30th November 2007

For further details contact Sally McClean (si.mcclean@ulster.ac.uk).

Seeing one self as others' see us

Adele kindly bought this model of an old doctor in America and gave it to me. I leave it to you to guess what the old doctor is going to do next.

However, the words on the plinth "Old doctor's never die, they only loose their patience" reminded me of a ward round in 1974. Surrounded as we were then, 'God going round with his angels' - junior doctors, medical students, nurses, therapists, I said to a lady of 104 ."You can go home on Monday and I will arrange, home help, district nurse and meals of wheels for you". She replied "Don't be such a stupid little boy. Monday is not convenient for me and I am perfectly capable of making my own social arrangements." Then with a sweet smile, she ended 'Thank you for getting me better.'

Nosokinetics News is the newsletter of the UK Nosokinetics Group

Nosokinetics is the science / subject of measuring and modelling the dynamic aspects of patient and client movement (flow) through health and social care systems. From the Greek, literally, *noso* (disease) and *kinetics* (movement).

The group collaborates to organise conferences and disseminates news of our and others research and practical use of modelling to enhance decision making in health and social care systems.

Thanks to IMS our web archive of full texts of submitted papers is at: http://www.iol.ie/~rjtechne/millard/index0.htm

Past issues in PDF at http://ww.nosokinetics.org/

To receive a personal copy follow the instructions at

http://www.jiscmail.ac.uk/lists/NOSOKINETICS-NEWSLETTER.html

Phase-type Distributions in Healthcare Modelling II

Mark Fackrell

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In the second of these three articles on phase-type (PH) distributions in healthcare modelling I will introduce Coxian distributions, a particularly useful subclass, and explain briefly how they have been used in healthcare modelling. In the first article (see Fackrell [3]) the case for using stochastic models in healthcare was made, and PH distributions were introduced. Fackrell [2] contains a more comprehensive treatment and bibliography.

An order p Coxian distribution has a representation of the form

$$\boldsymbol{\alpha} = \begin{pmatrix} \alpha_1 & \alpha_2 & \dots & \alpha_p \end{pmatrix} \tag{1}$$

$$\boldsymbol{T} = \begin{pmatrix} -\lambda_1 & \lambda_1 & 0 & \dots & 0 \\ 0 & -\lambda_2 & \lambda_2 & \dots & 0 \\ 0 & 0 & -\lambda_3 & \ddots & 0 \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & -\lambda_p \end{pmatrix}, \tag{2}$$

where $0 < \lambda_1 \leq \lambda_2 \leq \ldots \leq \lambda_p$. Figure 1 shows the state transition diagram for an order p Coxian distribution.

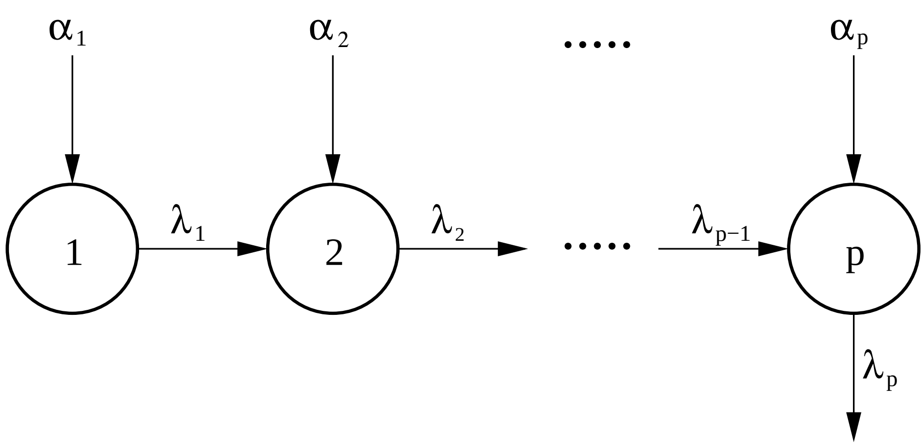

Figure 1: State transition diagram for an order p Coxian distribution.

The first thing we notice is that the representation (1)–(2) depends on only $2p-1$ parameters. General PH representations require p^2+p-1 parameters, but only $2p-1$ parameters are needed to define the distribution uniquely. Thus, general PH distributions are considerably overparameterized, whereas Coxian distributions are not. When it comes to fitting data with PH distributions, Coxian distributions have been the prefered option for practitioners in healthcare modelling and elsewhere, partly because of this reason - there are a lot less parameters to estimate.

Coxian distributions exhibit quite a lot of flexibility. Figure 2 shows the density functions for three order 4 Coxian distributions with the same generator $\boldsymbol{T}$, but different vectors $\boldsymbol{\alpha}$. We can see by their shapes that they exhibit much more flexibility than the exponential distribution, which should come as no surprise - they rely on 7 free parameters instead of just one! Indeed, Coxian distributions are more versatile than both hyperexponential distributions and generalized Erlang distributions, which also depend on $2p-1$ parameters, see Fackrell [2].

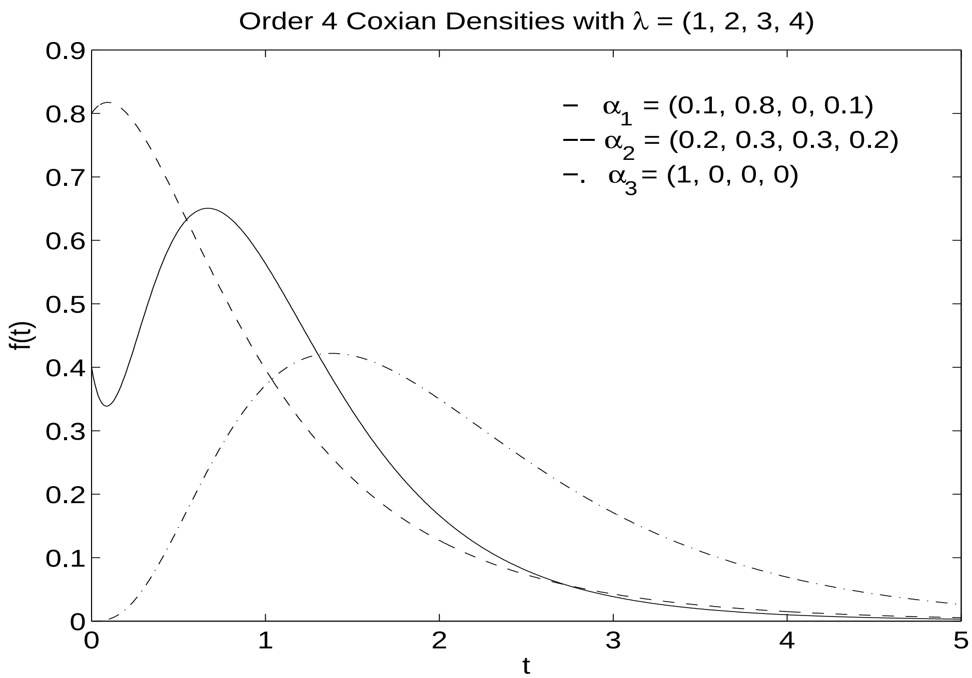

Figure 2: Density functions for three different order 4 Coxian distributions with $\lambda_1 = 1$, $\lambda_2 = 2$, $\lambda_3 = 3$, $\lambda_4 = 4$.

A curious mathematical fact is that any PH distribution whose generator $\boldsymbol{T}$ is an upper triangular matrix, has a Coxian representation of the same or lower order, see Cumani [1] or O'Cinneide [9]. For example, the hyperexponential distribution with representation

$$\boldsymbol{\alpha} = \begin{pmatrix} \frac{1}{6} & \frac{1}{2} & \frac{1}{3} \end{pmatrix} \tag{3}$$

$$\boldsymbol{T} = \begin{pmatrix} -2 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & -1 \end{pmatrix}, \tag{4}$$

has an equivalent Coxian representation

$$\boldsymbol{\beta} = \begin{pmatrix} \frac{1}{9} & \frac{1}{6} & \frac{13}{18} \end{pmatrix} \tag{5}$$

$$\boldsymbol{S} = \begin{pmatrix} -1 & 1 & 0 \\ 0 & -2 & 2 \\ 0 & 0 & -3 \end{pmatrix}, \tag{6}$$

In fact, any *PH* distribution whose generator $\boldsymbol{T}$ has only real eigenvalues has a Coxian representation of *some* order. The million dollar question is, of course, how big is the order? It is widely believed that there are examples of *PH* distributions of relatively low order that have Coxian representations of high order. Nevertheless, Coxian distributions are a very important subclass of *PH* distributions and need to be studied.

Coxian distributions have been popular with healthcare modellers because the states (or groups of states) can sometimes be given a physical interpretation. For example, Xie, Chaussalet, and Millard [10] modelled the length of stay (LOS) of geriatric patients in residential and nursing home care with two, 2-state Coxian distributions. Figure 3 shows the state transition diagram for their model. Here, patients enter the system via the

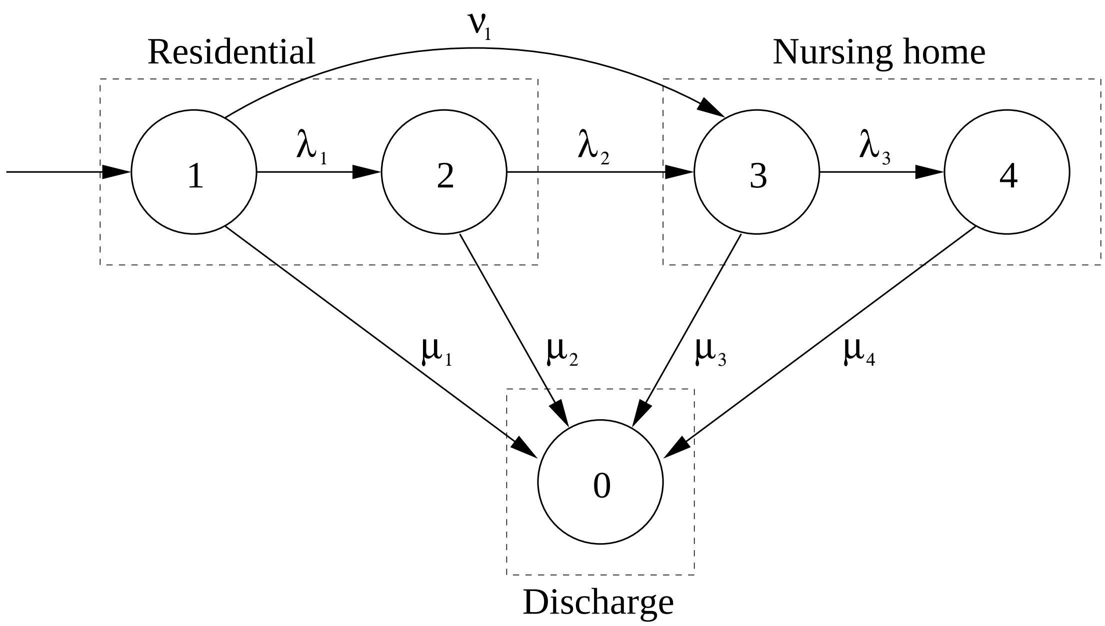

Figure 3: State transition diagram to model the length of stay in residential and nursing home care.

residential home care block where they can spend a short time (state 1 only), or a long time (state 1 followed by state 2). They can be discharged from either state, or progress to nursing home care, where again they can spend a short time (state 3), or a long time (states 3 and 4) before being discharged or dying.

The corresponding *PH* representation for the model is

$$\boldsymbol{\alpha} = \begin{pmatrix} 1 & 0 & 0 & 0 \end{pmatrix} \tag{7}$$

$$\boldsymbol{T} = \begin{pmatrix} -(\lambda_1 + \mu_1 + \nu_1) & \lambda_1 & \nu_1 & 0 \\ 0 & -(\lambda_2 + \mu_2) & \lambda_2 & 0 \\ 0 & 0 & -(\lambda_3 + \mu_3) & \lambda_3 \\ 0 & 0 & 0 & -\mu_4 \end{pmatrix}. \tag{8}$$

We remark here that because $\boldsymbol{T}$ is an upper triangular matrix, the *PH* distribution with representation $(\boldsymbol{\alpha}, \boldsymbol{T})$ is a Coxian distribution.

The authors fitted the Coxian distribution to four years data from the social services department of a London borough using maximum likelihood estimation. They reported

$\lambda_1 = \lambda_2 = \mu_2 = 0$, $\nu_1 = 0.000228$, $\mu_1 = 0.000855$, $\lambda_3 = 0.010874$, $\mu_3 = 0.006138$, and $\mu_4 = 0.001275$. The LOS in residential care was modelled by an exponential distribution (state 2 was unneccessary) with an average LOS of 923 days, with 21% of patients moving on to nursing home care and the rest getting discharged. The LOS for short stay patients in nursing home care (36 %) was modelled with an exponential distribution (average LOS 59 days), and the LOS for long stay patients (64 %) was modelled with a 2-state generalized Erlang distribution (average LOS 843 days).

Other notable papers where Coxian distributions have been used to model systems in healthcare include Faddy and McClean [4] and [5], Faddy and Taylor [6], McClean, Faddy, and Millard [8], and Marshall and McClean [7].

In the next issue's article I will propose some new ways in which *PH* distributions could be used in healthcare modelling.

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